

Fixed point for F_{\perp} -weak contraction

NEERAJ GARAKOTI*, MAHESH CHANDRA JOSHI, ROHIT KUMAR

ABSTRACT. In this paper, we establish some fixed point results for F_{\perp} -weak contraction in orthogonal metric space and we give an application for the solution of second order differential equation.

1. INTRODUCTION

In 2017, Gordeji et al. [3] introduced the idea of orthogonal metric spaces and gave the following definitions.

Definition 1 ([3]). Let X be a non-empty set and $\perp \subseteq X \times X$ be a binary relation. If \perp satisfies the condition

there exists x_0 ; ((for all y ; $y \perp x_0$) or (for all y ; $x_0 \perp y$)),

then (X, \perp) is called an orthogonal set (briefly O -set).

Example 1 ([3]). Let X be the collection of people in world. Define orthogonality as $x \perp y$ if x can donate blood to y . As we know blood group AB^+ is universal acceptor and O^- is universal donor so X is an O -set where orthogonal element is not unique.

Type	You can give blood to	You can receive blood to
A^+	A^+, AB^+	A^+, A^-, O^+, O^-
O^+	O^+, A^+, B^+, AB^+	O^+, O^-
B^+	B^+, AB^+	B^+, B^-, O^+, O^-
AB^+	AB^+	<i>Everyone</i>
A^-	A^-, A^+, AB^+, AB^-	A^-, O^-
O^-	<i>Everyone</i>	O^-
B^-	B^-, B^+, AB^+, AB^-	B^-, O^-
AB^-	AB^-, AB^+	AB^-, B^-, O^-, A^-

2020 *Mathematics Subject Classification*. Primary: 47H10; Secondary: 54H25.

Key words and phrases. Orthogonal metric space, F_{\perp} -contraction, F_{\perp} -weak contraction.

Full paper. Received 2 June 2020, accepted 19 February 2021, available online 16 March 2021.

Author thanks UGC, New Delhi, for support under NET-JRF.

Definition 2 ([3]). A sequence $\{x_n\}_{n \in \mathbb{N}}$ in O -set is called orthogonal sequence (briefly O -sequence) if either

$$x_n \perp x_{n+1} \text{ or } x_{n+1} \perp x_n, \text{ for all } n \in \mathbb{N}.$$

Definition 3 ([3]). Let (X, d, \perp) is said to be an orthogonal metric space if (X, \perp) is an O -set and (X, d) is a metric space. The space X is orthogonally complete (briefly O -complete) if every Cauchy O -sequence is convergent.

Definition 4 ([3]). Let (X, d, \perp) be an orthogonal metric space.

- (1) A mapping $f : X \rightarrow X$ is said to be orthogonal contraction (\perp -contraction) if

$$d(fx, fy) \leq \lambda d(x, y), \text{ for all } x \perp y \text{ and } 0 < \lambda < 1.$$

- (2) A mapping $f : X \rightarrow X$ is called orthogonal preserving (\perp -preserving) mapping if $x \perp y$ then $f(x) \perp f(y)$, and f is said to be \perp -weakly preserving if $f(x) \perp f(y)$ or $f(y) \perp f(x)$ whenever $x \perp y$.
- (3) A mapping $f : X \rightarrow X$ is orthogonal continuous (\perp -continuous) if for each O -sequence $\{a_n\}_{n \in \mathbb{N}}$ in X whenever $a_n \rightarrow a$ implies $f(a_n) \rightarrow f(a)$ as $n \rightarrow \infty$. Also, f is \perp -continuous if f is \perp -continuous for each $a \in X$.

Generalizing Banach contraction, in 2012, Wardowski [8] introduced a new concept of contraction called F -contraction. Thereafter, F -contraction has been extended in different settings and various spaces with their applications, see [1, 2, 4, 7].

Definition 5 ([8]). Let \mathcal{F} be the family of functions $F : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that

- (F1) F is strictly increasing, i.e. for all $\alpha, \beta \in \mathbb{R}^+$ such that $\alpha < \beta \implies F(\alpha) < F(\beta)$;
- (F2) For each sequence $\{\alpha_n\}_{n \in \mathbb{N}}$ of positive numbers $\lim_{n \rightarrow \infty} \alpha_n = 0$ if and only if $\lim_{n \rightarrow \infty} F(\alpha_n) = -\infty$;
- (F3) There exists $j \in (0, 1)$ such that $\lim_{\alpha \rightarrow 0^+} \alpha^j F(\alpha) = 0$.

A mapping $T : X \rightarrow X$ is said to be an F -contraction if there exists $\tau > 0$ such that, for all $x, y \in X$,

$$(1) \quad d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(d(x, y)).$$

Example 2 ([8]). Let $F : \mathbb{R}_+ \rightarrow \mathbb{R}$ be given by a formula $F(\alpha) = \ln(\alpha)$. It is clear that F satisfies (F1 – F3) for any $j \in (0, 1)$. Each mapping $T : X \rightarrow X$ satisfying condition (1) is an F -contraction such that

$$d(Tx, Ty) \leq e^{-\tau} d(x, y), \quad \text{for all } x, y \in X, Tx \neq Ty.$$

It is clear that for $x, y \in X$ such that $Tx = Ty$, the inequalities $d(Tx, Ty) \leq e^{-\tau} d(x, y)$ also holds, i.e. T is Banach contraction.

Using the concept of F -contraction, Wardowski [8] generalized the Banach Contraction Principle and proved that every F -contraction in a complete metric space has a unique fixed point. Later in 2014, Wardowski [9]

introduced the concept of F -weak contraction and established some fixed point results.

Recently, Sawangsup et al. [6] generalized F -contraction in orthogonal metric space and proved some fixed point results for F_{\perp} -contraction mappings.

Definition 6 ([6]). A map $T : X \rightarrow X$ is said to be orthogonal F -contraction or F_{\perp} -contraction on an O -metric space (X, \perp, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $x \perp y$,

$$(2) \quad d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(d(x, y)).$$

2. MAIN RESULTS

Now, we generalize the condition of Wardowski [9] and prove some results in orthogonal metric spaces.

Definition 7. A map $T : X \rightarrow X$ is said to be orthogonal F -weak contraction on an O -metric space (X, \perp, d) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $x \perp y$,

$$(3) \quad d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(M(x, y)),$$

where

$$M(x, y) = \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\}.$$

Theorem 1. Let (X, \perp, d) be an O -complete metric space and T be a \perp -preserving, F_{\perp} -weak contraction, \perp -continuous on X . Then, T has unique fixed point in X .

Proof. From the definition of orthogonality, it follows that $x_0 \perp f(x_0)$ or $f(x_0) \perp x_0$. Define a sequence $x_{n+1} = Tx_n$, for all $n \in \mathbb{N}_0$ ($\mathbb{N}_0 = \mathbb{N} \cup \{0\}$). If there exist $m \in \mathbb{N}_0$ such that $Tx_{m+1} = Tx_m$, then x_m is fixed point of T . Thus we assume that $x_{n+1} \neq x_n$ for all $n \in \mathbb{N}_0$.

Since, T is \perp -preseving, we have

$$\text{either } x_n \perp x_{n+1} \text{ or } x_{n+1} \perp x_n,$$

for all $n \in \mathbb{N}_0$. This implies that $\{x_n\}$ is an O -sequence. By equation (3), we have

$$\begin{aligned} d(x_n, x_{n+1}) &= d(Tx_{n-1}, Tx_n) > 0 \implies \\ &\tau + F(d(Tx_{n-1}, Tx_n)) \leq F(M(x_{n-1}, x_n)), \end{aligned}$$

where

$$M(x_{n-1}, x_n) = \max \left\{ d(x_{n-1}, x_n), d(x_{n-1}, Tx_{n-1}), d(x_n, Tx_n), \right.$$

$$\begin{aligned}
& \left. \frac{d(x_n, Tx_{n-1}) + d(x_{n-1}, Tx_n)}{2} \right\} \\
= & \max \left\{ d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_{n+1})}{2} \right\} \\
\leq & \max \left\{ d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{2} \right\} \\
= & \max \left\{ d(x_{n-1}, x_n), d(x_n, x_{n+1}) \right\}.
\end{aligned}$$

If $\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_n, x_{n+1})$, for some n , using equation (3), we have

$$F(d(x_n, x_{n+1})) \leq F(d(x_n, x_{n+1})) - \tau < F(d(x_n, x_{n+1})),$$

which is a contradiction. Therefore, for all $n \in \mathbb{N}$, we have

$$F(d(x_n, x_{n+1})) \leq F(d(x_{n-1}, x_n)) - \tau.$$

Hence, we get

$$(4) \quad F(d(x_n, x_{n+1})) \leq F(d(x_0, x_1)) - n\tau, \quad \text{for all } n \in \mathbb{N}.$$

Taking limit as $n \rightarrow \infty$, we get

$$\lim_{n \rightarrow \infty} F(d(x_n, x_{n+1})) = -\infty.$$

Hence, by the (F2) property

$$(5) \quad \lim_{n \rightarrow \infty} d(x_n, x_{n+1}) = 0.$$

By property (F3), there exist $j \in (0, 1)$ such that

$$(6) \quad \lim_{n \rightarrow \infty} (d(x_n, x_{n+1}))^j F(d(x_n, x_{n+1})) = 0.$$

Multiplying (4) by $d(x_n, x_{n+1})^j$, we get

$$(d(x_n, x_{n+1}))^j (F(d(x_n, x_{n+1})) - F(d(x_0, x_1))) \leq -n\tau (d(x_n, x_{n+1}))^j \leq 0.$$

Taking limit and using equations (5) and (6), we get

$$\lim_{n \rightarrow \infty} \left(n(d(x_n, x_{n+1}))^j \right) = 0.$$

Then, there exists $n_1 \in \mathbb{N}$ s.t. $n(d(x_n, x_{n+1}))^j \leq 1$, for all $n \geq n_1$, we get

$$(7) \quad d(x_n, x_{n+1}) \leq \frac{1}{n^{\frac{1}{j}}}.$$

Using triangular inequality and equation (7), for all $m, n \in \mathbb{N}$ s.t. $m > n \geq n_1$, we get

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &\leq \sum_{i=n}^{\infty} d(x_i, x_{i+1}) \leq \sum_{i=n}^{\infty} \frac{1}{i^j}. \end{aligned}$$

The series $\sum_{i=n}^{\infty} \frac{1}{i^j}$ is convergent. It follows that $\{x_n\}$ is a Cauchy O -sequence in X . Since X is O -complete, there exists $z \in X$ such that $x_n \rightarrow z$ as $n \rightarrow \infty$. Since T is \perp -continuous, we have

$$Tz = T(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} x_{n+1} = z.$$

Hence, z is a fixed point of T .

For the uniqueness of fixed point, if w is another fixed point of T , then from equation (3), we get

$$\begin{aligned} \tau + F(d(w, z)) &= \tau + F(d(Tw, Tz)) \\ &\leq F\left(\max\left\{d(w, z), d(w, Tw), d(z, Tz), \right. \right. \\ &\quad \left. \left. \frac{d(w, Tz) + d(z, Tw)}{2}\right\}\right) - \tau \\ &= F(d(w, z)), \end{aligned}$$

a contradiction that $\tau \leq 0$. So, $d(w, z) = 0$, i.e., $w = z$. \square

Example 3. Let $X = [0, 1] \cap \mathbb{Q}$ and the metric is defined by $d(x, y) = |x - y|$. Define the binary relation \perp on X by $x \perp y$ if $xy = 0$ or x . It is easy to see that $0 \perp y$ for all $y \in X$. Hence, (X, \perp) is an O -set. As we know the set of \mathbb{Q} with Euclidian metric is not a complete metric space. Hence, X is not a complete metric space but it is O -complete.

Let T be a map defined as

$$T(x) = \begin{cases} \frac{x}{2}, & \text{if } x \in [0, 1) \cap \mathbb{Q}; \\ 0, & \text{if } x = 1. \end{cases}$$

It is easy to see that T is \perp -continuous and \perp -preserving. But, T is not F_{\perp} -contraction because for $x = 1$ and $y = 3/4$, where 1 is orthogonal for each $y \in [0, 1] \cap \mathbb{Q}$,

$$d(x, y) = d(1, \frac{3}{4}) = \frac{1}{4} < \frac{3}{8} = d(T1, T\frac{3}{4}) = d(Tx, Ty).$$

Hence, by equation (2), we get $\tau < 0$ which is contradiction.

Now, we prove that T satisfies the condition of F_{\perp} -weak contraction. Since, 0 is orthogonal for each $y \in [0, 1] \cap \mathbb{Q}$. Therefore, for $x = 0$ and $y \in (0, 1) \cap \mathbb{Q}$,

$$\begin{aligned} d(T0, Ty) &= d(0, \frac{y}{2}) = \frac{y}{2} < y = d(0, y) \\ &= \max \left\{ d(0, y), d(0, T0), d(y, Ty), \frac{d(0, Ty) + d(y, T0)}{2} \right\}. \end{aligned}$$

Also, 1 is orthogonal for each $y \in [0, 1] \cap \mathbb{Q}$. For $x = 1$ and $y \in (0, 1) \cap \mathbb{Q}$

$$\begin{aligned} d(T1, Ty) &= d(0, \frac{y}{2}) = \frac{y}{2} < 1 = d(1, T1) \\ &= \max \left\{ d(1, y), d(1, T1), d(y, Ty), \frac{d(1, Ty) + d(y, T1)}{2} \right\}. \end{aligned}$$

Hence, T is F_{\perp} -weak contraction for $F(x) = \ln x$ and $\tau = \ln 2$. By Theorem (1), T has a unique fixed point, i.e., $x = 0$. \square

Remark 1. We get the result of Sawangsup et al. [6] as following corollary.

Corollary 1. *Let (X, \perp, d) be an O -complete metric space and $T : X \rightarrow X$ is an F_{\perp} -contraction with \perp -preserving and \perp -continuous. Then T has a unique fixed point $z \in X$.*

Corollary 2. *Let (X, \perp, d) be an O -complete metric space and $T : X \rightarrow X$. If there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, T is \perp -continuous, \perp -preserving and T satisfies*

$$\begin{aligned} d(Tx, Ty) > 0 &\Rightarrow \tau + F(d(Tx, Ty)) \\ &\leq F\left(\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \right. \\ &\quad \left. \delta[d(x, Ty) + d(y, Tx)]\right), \end{aligned}$$

for all $x, y \in X$ with $x \perp y$, where $\alpha, \beta, \gamma, \delta \geq 0$ and $\alpha + \beta + \gamma + 2\delta < 1$. Then T has a unique fixed point $z \in X$.

Proof. For all $x, y \in X$, with $x \perp y$, we have

$$\begin{aligned} &\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \delta[d(x, Ty) + d(y, Tx)] \\ &\leq (\alpha + \beta + \gamma + 2\delta) \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\} \\ &\leq \max \left\{ d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2} \right\}. \end{aligned}$$

Following the proof of theorem (1) we get the required result. \square

The following result is generalization of the result given in [5] where the existence of the fixed point for the mappings satisfying a contraction condition for the points of a closed ball was considered.

Theorem 2. Let (X, \perp, d) be an O -complete metric space and $T : X \rightarrow X$. If T is F_{\perp} -weak contraction for all $x, y \in \overline{B(x_0, r)}$, \perp -preserving and \perp -continuous on X . Moreover, for any $r > 0$,

$$\sum_{j=0}^{\mathbb{N}} d(x_j, Tx_j) \leq r, \text{ for all } j \in \mathbb{N}.$$

Then there exists a point $z \in \overline{B(x_0, r)}$ s.t. $Tz = z$.

Proof. We construct a sequence x_n such that $x_n = Tx_{n-1}$ and show that it is O -Cauchy sequence. First we claim that $x_n \in \overline{B(x_0, r)}$ for all n . We have,

$$d(x_0, x_1) = d(x_0, Tx_0) \leq r.$$

Suppose, $x_2, x_3, \dots, x_n \in \overline{B(x_0, r)}$. Following the proof of Theorem (1), we have

$$F(d(x_n, x_{n+1})) \leq F(d(x_{n-1}, x_n)) - \tau.$$

Hence, by property (F1), $d(x_{n+1}, x_n) < d(x_{n-1}, x_n)$. Thus,

$$\begin{aligned} d(x_0, x_{n+1}) &\leq d(x_0, x_1) + d(x_1, x_2) + \dots + d(x_n, x_{n+1}) \\ &= \sum_{j=0}^{\mathbb{N}} d(x_j, Tx_j) \leq r \implies x_{n+1} \in \overline{B(x_0, r)}. \end{aligned}$$

Hence, $x_n \in \overline{B(x_0, r)}$, for all $n \in \mathbb{N}$. Again, following the proof of theorem (1), sequence $\{x_n\}$ is O -Cauchy sequence and $\{x_n\} \rightarrow z \in \overline{B(x_0, r)}$. Since T is \perp -continuous, we have $Tz = z$. \square

Remark 2. Taking (X, d) a complete metric space in Theorem 2, we get the following result.

Corollary 3. Let (X, d) be a complete metric space, $T : X \rightarrow X$ be a mapping, $r > 0$ and x_0 be an arbitrary point in X . Suppose there exists $k \in [0, 1)$ with

$$d(Tx, Ty) \leq k d(x, y), \quad \text{for all } x, y \in \overline{B(x_0, r)},$$

and $d(x_0, Tx_0) < (1 - k)r$. Then there exists a unique point $z \in \overline{B(x_0, r)}$ such that $z = Tz$.

3. APPLICATIONS

Now, we apply our result to prove the existence theorem for the solution of the second order differential equation. which represents motion of spring under exterior force (for more details see [7]) and given by;

$$(8) \quad \begin{cases} \frac{d^2 u}{dt^2} + \frac{c}{m} \frac{du}{dt} = K(t, (u(t))); \\ u(0) = 0, \quad u'(0) = a, \end{cases}$$

where $K : [0, I] \times \mathbb{R}^+ \rightarrow \mathbb{R}$ is a continuous function and $I > 0$. Above problem is equivalent to the integral equation

$$(9) \quad u(t) = \int_0^t G(t, s)K(s, u(s)) \, ds, \quad t \in [0, I],$$

where $G(t, s)$ is the Green's function, given by

$$G(t, s) = \begin{cases} (t-s)e^{\tau(t-s)}, & 0 \leq s \leq t \leq I; \\ 0, & 0 \leq t \leq s \leq I. \end{cases}$$

where $\tau > 0$ is a constant, calculated in terms of c and m .

Let $X = C([0, I], \mathbb{R}^+)$ be the set of all non negative continuous real functions defined on $[0, I]$ with

$$u \perp v \iff u(t)v(t) \geq 0,$$

for an arbitrary $u, v \in X$. We define

$$\|u\|_{\tau} = \sup_{t \in [0, I]} \{ |u(t)|e^{-2\tau t} \}, \quad \text{where } \tau > 0.$$

Hence, (X, d) is an O -complete metric space. Consider the self-map $T : X \rightarrow X$ defined as

$$Tu(t) = \int_0^t G(t, s)K(s, u(s)) \, ds, \quad t \in [0, I].$$

Clearly, if equation (8), possess a solution then it must be fixed point of T .

Theorem 3. *Suppose:*

- (i) K is increasing function,
- (ii) $u \perp v \iff u(t)v(t) \geq 0$,
- (iii) there exists $\tau > 0$ such that

$$|K(s, u) - K(s, v)| \leq \tau^2 e^{-\tau} M(u, v),$$

for all $s \in [0, I], u, v \in \mathbb{R}^+$, where

$$M(u, v) = \max \left\{ |u - v|, |u - Tu|, |v - Tv|, \frac{|u - Tv| + |v - Tu|}{2} \right\},$$

such that

$$M_{\tau}(u, v) = \sup_{t \in [0, I]} \{ e^{-2\tau t} M(u, v) \}.$$

Then the equation (9) has a unique solution.

Proof. Since $X = C([0, I], \mathbb{R}^+)$ is an O -complete metric space. It is clear that T is \perp -continuous and \perp -preserving.

Now, for all $u, v \in X$ with $u \perp v$ and $Tu(t) \neq Tv(t)$, we have

$$\begin{aligned}
|Tu(t) - Tv(t)| &\leq \int_0^t G(t, s) |K(s, u(s)) - K(s, v(s))| \, ds \\
&\leq \int_0^t G(t, s) \tau^2 e^{-\tau} M(u, v) \, ds \\
&= \int_0^t G(t, s) \tau^2 e^{-\tau} e^{2\tau s} e^{-2\tau s} M(u, v) \, ds \\
&\leq \tau^2 e^{-\tau} M_\tau(u, v) \int_0^t G(t, s) e^{2\tau s} \, ds \\
&= \tau^2 e^{-\tau} M_\tau(u, v) \int_0^t e^{2\tau s} (t-s) e^{\tau(t-s)} \, ds \\
&= \tau^2 e^{-\tau} M_\tau(u, v) e^{\tau t} \int_0^t e^{\tau s} (t-s) \, ds \\
&= \tau^2 e^{-\tau} M_\tau(u, v) e^{\tau t} \times \frac{e^{\tau t}}{\tau^2} (1 - \tau t e^{-\tau t} - e^{-\tau t}) \\
&\leq e^{-\tau} M_\tau(u, v) e^{2\tau t}; \quad (1 - \tau t e^{-\tau t} - e^{-\tau t}) \leq 1.
\end{aligned}$$

Therefore,

$$\begin{aligned}
|Tu(t) - Tv(t)| e^{-2\tau t} &\leq e^{-\tau} M_\tau(u, v), \\
\|Tu - Tv\|_\tau &\leq e^{-\tau} M_\tau(u, v).
\end{aligned}$$

Taking logarithms, we get

$$\begin{aligned}
\ln(\|Tu - Tv\|_\tau) &\leq \ln(e^{-\tau} M_\tau(u, v)), \\
\tau + \ln(\|Tu - Tv\|_\tau) &\leq \ln M_\tau(u, v).
\end{aligned}$$

T satisfies all the criteria for $F(x) = \ln(x)$ of Theorem 1. Hence, we get a unique solution of equation (9). \square

REFERENCES

- [1] H. Ayadi, E. Karapinar and H. Yazidi, *Modified F - contraction via α -admissible mappings and application to integral equation*, Filomat, 5 (2017), 1141-1148.
- [2] H. Baghani, M.E. Gordji and M. Rameani, *Orthogonal sets: The axiom of choice and proof of a fixed point theorem*, Journal of Fixed Point Theory and Applications, 18 (2016), 465-477.
- [3] M.E. Gordji, M. Rameani, M. De La Sen, and Y.J. Cho, *On orthogonal sets and Banach fixed point theorem*, Fixed Point Theory, 18 (2017), 569-578.
- [4] A. Hussain and M. Arshad, *New type of multivalued F -Contraction involving Fixed Point on Closed Ball*, Journal of Mathematics and Computer Science (JMCS), 17 (2) (2017), 246-254.
- [5] E. Kryszewicz, *Introductory functional analysis with applications*, John Wiley & Sons, New York, 1989.

- [6] K. Sawangsup, W. Sintunavarat and Y.J. Cho, *Fixed point theorems for orthogonal F -contraction mappings on O -complete metric space*, Journal of Fixed Point Theory and Applications, 22 (2020).
- [7] D. Singh, V. Joshi, M. Imdad and P. Kumam, *Fixed point theorems via generalized F -contractions with applications to functional equations occurring in dynamic programming*, Journal of Fixed Point Theory and Applications, 19 (2017), 1453-1479.
- [8] D. Wardowski, *Fixed point theory of a new type of contractive mappings in complete metric spaces*, Fixed Point Theory and Applications, 2012 (2012), Article ID: 94, 6 pages.
- [9] D. Wardowski and N. Van Dung, *Fixed point of F -weak contractions on complete metric spaces*, Demonstratio Mathematica, 47 (2014), 146-155.

NEERAJ GARAKOTI

DEPARTMENT OF MATHEMATICS
D.S.B. CAMPUS, KUMAUN UNIVERSITY
NAINITAL-263002
INDIA

E-mail address: neerajgarakoti@gmail.com

MAHESH CHANDRA JOSHI

DEPARTMENT OF MATHEMATICS
D.S.B. CAMPUS, KUMAUN UNIVERSITY
NAINITAL-263002,
INDIA

E-mail address: mcjoshi69@gmail.com

ROHIT KUMAR

DEPARTMENT OF MATHEMATICS
D.S.B. CAMPUS, KUMAUN UNIVERSITY
NAINITAL-263002
INDIA

E-mail address: rohitk0351@gmail.com